DM-02-03 ► 設 $A_n = [0, 1 - \frac{1}{n}],\ n = 1, 2, 3, \ldots$，試求 $\bigcup_{n=1}^{\infty} A_n$。

【解】 因

$$
\begin{aligned}
A_1 &= [0, 1 - \frac{1}{1}] = [0, 0] \\
A_2 &= [0, 1 - \frac{1}{2}] = [0, \frac{1}{2}] \\
A_3 &= [0, 1 - \frac{1}{3}] = [0, \frac{2}{3}] \\
&\vdots \\
A_k &= [0, 1 - \frac{1}{k}] = [0, \frac{k-1}{k}] \\
&\vdots
\end{aligned}
$$

且 $A_k \subset A_{k+1}$，$k = 1, 2, 3, \ldots$，故

$$
\bigcup_{n=1}^{\infty} A_n = \lim_{n\to\infty} A_n = \lim_{n\to\infty} [0, \frac{n-1}{n}] = [0, 1]
$$

□