

DM-02-03 ▶ 設 $A_n = (0, 1 + \frac{1}{n})$, $n = 1, 2, 3, \dots$, 試求 $\bigcup_{n=1}^{\infty} A_n$ 。

【解】 因

$$A_1 = (0, 1 + \frac{1}{1}) = (0, 2)$$

$$A_2 = (0, 1 + \frac{1}{2}) = (0, \frac{3}{2})$$

$$A_3 = (0, 1 + \frac{1}{3}) = (0, \frac{4}{3})$$

⋮

$$A_k = (0, 1 + \frac{1}{k}) = (0, \frac{k+1}{k})$$

⋮

且 $A_{k+1} \subset A_k$, $k = 1, 2, 3, \dots$, 故

$$\bigcup_{n=1}^{\infty} A_n = A_1 = (0, 2)$$

□