▶ **Problem DM-3.3-12** Show that composition of relation is an associative operation. That is, show that if R, S, and T are binary relation on a set X, then

$$R \circ (S \circ T) = (R \circ S) \circ T$$

Proof. Let $(x, w) \in X^2$. We will show that $(x, w) \in R \circ (S \circ T)$ if and only if $(x, w) \in (R \circ S) \circ T$. The proof is as follows:

$$\begin{aligned}
& (x, w) \in (R \circ S) \circ T \\
\Leftrightarrow\ & \text{there exists an element } y \in X \text{ such that } (x, y) \in T \text{ and } (y, w) \in R \circ S \\
\Leftrightarrow\ & \text{there exist elements } y, z \in X \text{ such that } (x, y) \in T, (y, z) \in S, \text{ and } (z, w) \in R \\
\Leftrightarrow\ & \text{there exists an element } z \in X \text{ such that } (x, z) \in S \circ T \text{ and } (z, w) \in R \\
\Leftrightarrow\ & (x, z) \in R \circ (S \circ T)
\end{aligned}$$

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