▶ **Problem DM-3.3-14** Let $X = \{0, 1\}$. Let $B = \mathcal{P}(X \times X)$ be the set of all binary relations on $X$.

(a) List all the element of $B$.

(b) Since elements of $B$ are themselves relations, it makes sence to ask whether two of those relations are inverses of each other. Let

$$\textit{IsInverseOf} = \{(R, S) : R \in B \text{ and } S \in B \text{ and } R = S^{-1}\}$$

List all elements of *IsInverseOf*.

**Solution.** (a) Since $X = \{0, 1\}$, we have $X \times X = \{(0, 0), (0, 1), (1, 0), (1, 1)\}$. Thus

$$\begin{aligned}
B &= \mathcal{P}(X \times X) = \mathcal{P}(\{(0,0), (0,1), (1,0), (1,1)\}) \\
&= \{\emptyset, \{(0,0)\}, \{(0,1)\}, \{(1,0)\}, \{(1,1)\}, \{(0,0), (0,1)\}, \{(0,0), (1,0)\}, \\
&\quad \{(0,0), (1,1)\}, \{(0,1), (1,0)\}, \{(0,1), (1,1)\}, \{(1,0), (1,1)\}, \\
&\quad \{(0,0), (0,1), (1,0)\}, \{(0,0), (0,1), (1,1)\}, \{(0,0), (1,0), (1,1)\}, \\
&\quad \{(0,1), (1,0), (1,1)\}, \{(0,0), (0,1), (1,0), (1,1)\}\}
\end{aligned}$$

(b) By the definition of inverse, we have
$\emptyset^{-1} = \emptyset$, $\{(0,0)\}^{-1} = \{(0,0)\}$, $\{(0,1)\}^{-1} = \{(1,0)\}$, $\{(1,0)\}^{-1} = \{(0,1)\}$, $\{(1,1)\}^{-1} = \{(1,1)\}$,
$\{(0,0), (0,1)\}^{-1} = \{(0,0), (1,0)\}$, $\{(0,0), (1,0)\}^{-1} = \{(0,0), (0,1)\}$,
$\{(0,0), (1,1)\}^{-1} = \{(0,0), (1,1)\}$, $\{(0,1), (1,0)\}^{-1} = \{(0,1), (1,0)\}$,
$\{(0,1), (1,1)\}^{-1} = \{(1,0), (1,1)\}$, $\{(1,0), (1,1)\}^{-1} = \{(0,1), (1,1)\}$,
$\{(0,0), (0,1), (1,0)\}^{-1} = \{(0,0), (0,1), (1,0)\}$, $\{(0,0), (0,1), (1,1)\}^{-1} = \{(0,0), (1,0), (1,1)\}$,
$\{(0,0), (1,0), (1,1)\}^{-1} = \{(0,0), (0,1), (1,1)\}$, $\{(0,1), (1,0), (1,1)\}^{-1} = \{(0,1), (1,0), (1,1)\}$,
$\{(0,0), (0,1), (1,0), (1,1)\}^{-1} = \{(0,0), (0,1), (1,0), (1,1)\}$

$$\begin{aligned}
\textit{IsInverseOf} &= \{(\emptyset, \emptyset), \\
&\quad (\{(0,0)\}, \{(0,0)\}), (\{(0,1)\}, \{(1,0)\}), (\{(1,0)\}, \{(0,1)\}), (\{(1,1)\}, \{(1,1)\}), \\
&\quad (\{(0,0), (0,1)\}, \{(0,0), (1,0)\}), (\{(0,0), (1,0)\}, \{(0,0), (0,1)\}), \\
&\quad (\{(0,0), (1,1)\}, \{(0,0), (1,1)\}), (\{(0,1), (1,0)\}, \{(0,1), (1,0)\}), \\
&\quad (\{(0,1), (1,1)\}, \{(1,0), (1,1)\}), (\{(1,0), (1,1)\}, \{(0,1), (1,1)\}), \\
&\quad (\{(0,0), (0,1), (1,0)\}, \{(0,0), (0,1), (1,0)\}), \\
&\quad (\{(0,0), (0,1), (1,1)\}, \{(0,0), (1,0), (1,1)\}), \\
&\quad (\{(0,0), (1,0), (1,1)\}, \{(0,0), (0,1), (1,1)\}), \\
&\quad (\{(0,1), (1,0), (1,1)\}, \{(0,1), (1,0), (1,1)\}), \\
&\quad (\{(0,0), (0,1), (1,0), (1,1)\}, \{(0,0), (0,1), (1,0), (1,1)\})\}
\end{aligned}$$

□