► **Problem DM-3.3-14** Let $X = \{0, 1\}$. Let $B = \mathcal{P}(X \times X)$ be the set of all binary relations on X.

(a) List all the element of B.

(b) Since elements of B are themselves relations, it makes sence to ask whether two of those relations are inverses of each other. Let

$$\mathit{IsInverseOf} = \{(R, S) : R \in B \text{ and } S \in B \text{ and } R = S^{-1}\}$$

List all elements of *IsInverseOf*.

(c) Since *IsInverseOf* is a binary relation, it has an inverse. What is $\mathit{IsInverseOf}^{-1}$?

(d) What is $\mathit{IsInverseOf} \circ \mathit{IsInverseOf}$?

Solution. (c) It is easy to see from (b) that $\mathit{IsInverseOf}^{-1} = \mathit{IsInverseOf}$.

(d)

$$\begin{aligned}
&\mathit{IsInverseOf} \circ \mathit{IsInverseOf} = \\
&\{(\emptyset, \emptyset), \\
&(\{(0,0)\}, \{(0,0)\}), (\{(0,1)\}, \{(0,1)\}), (\{(1,0)\}, \{(1,0)\}), (\{(1,1)\}, \{(1,1)\}), \\
&(\{(0,0),(0,1)\}, \{(0,0),(0,1)\}), (\{(0,0),(1,0)\}, \{(0,0),(1,0)\}), \\
&(\{(0,0),(1,1)\}, \{(0,0),(1,1)\}), (\{(0,1),(1,0)\}, \{(0,1),(1,0)\}), \\
&(\{(0,1),(1,1)\}, \{(0,1),(1,1)\}), (\{(1,0),(1,1)\}, \{(1,0),(1,1)\}), \\
&(\{(0,0),(0,1),(1,0)\}, \{(0,0),(0,1),(1,0)\}), \\
&(\{(0,0),(0,1),(1,1)\}, \{(0,0),(0,1),(1,1)\}), \\
&(\{(0,0),(1,0),(1,1)\}, \{(0,0),(1,0),(1,1)\}), \\
&(\{(0,1),(1,0),(1,1)\}, \{(0,1),(1,0),(1,1)\}), \\
&(\{(0,0),(0,1),(1,0),(1,1)\}, \{(0,0),(0,1),(1,0),(1,1)\})\}
\end{aligned}$$