▶ **Problem DM-3.5-18** Is there a reasonable notion of antisymmetric closure? Why, or why not?

Solution. Let R be a binary relation on a set X. For a particular property $\mathscr{P}$, the $\mathscr{P}$-*closure* of R is defined to be the smallest relation R' that contains R and has the desired property $\mathscr{P}$. From this concept, we will show that it has no any significance about "antisymmetric closure". By the definition that R is antisymmetric, it means that for any $x, y \in X$ and $x \neq y$, if $(x, y) \in R$ then $(y, x) \notin R$. We now consider a certain relation R which is not antisymmetric. That is, there exist $x, y \in X$ and $x \neq y$ such that $(x, y) \in R$ and $(y, x) \in R$. Obviously, if there is a binary relation R' containing R, then $(x, y) \in R'$ and $(y, x) \in R'$. Thus, we cannot find any binary relation R' containing R that satisfies the antisymmetric property.

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