▶ **Problem DM-3.7-14** There is an old, fallacious proof that if a relation is both symmetric and transitive, it is reflexive. We give this "proof" below. What is the error?

> Suppose R is a symmetric and transitive relation on a set X. Pick an $x \in X$. We need to show xRx. So, take any y where xRy. By symmetry, it follows that yRx. By transitivity, it follows that xRx.

Solution. Let R be a symmetric and transitive relation on s set X. To show that R is reflexive (i.e., xRx for every $x \in X$), the proof is based on the *assumption* that there is an element $y \in X$ such that xRy. However, this is not true for any symmetric and transitive relation R. For example, consider $X = \{1, 2, 3\}$ and $R = \{(1,1), (1,2), (2,1), (2,2)\}$. It is clear that R is symmetric and transitive on X, but R is not reflexive since $(3,3) \notin R$.

□