▶ **Problem DM-3.7-16(b)** Problem DM-3.7-16(a) says that if we take two equivalence classes $[m]$ and $[n]$, then we can unambiguously define $[m]+[n]$ and $[m]\cdot[n]$. Pick any $m_1 \in [m]$ and any $n_1 \in [n]$, and define

$$[m]+[n] \equiv [m_1+n_1]$$

and

$$[m]\cdot[n] \equiv [m_1 \cdot n_1]$$

The definition is unambiguous since it doesn't matter which m_1 and n_1 we pick. Find the addition and multiplication tables for the equivalence classes of $\equiv \pmod 4$ and $\equiv \pmod 5$. (Hint: for both $\equiv \pmod 4$ and $\equiv \pmod 5$, your answer should include

$$[0]+[0] \equiv [0], [0]+[1] \equiv [1], [0]\cdot[0] \equiv [0]$$

and

$$[1]\cdot[1] \equiv [1]$$

but, for $\equiv \pmod 4$,

$$[2]+[2] \equiv [0]$$

whereas, that will be false for $\equiv \pmod 5$.)

Proof.

The addition and multiplication tables for the equivalence classes of $\equiv \pmod 4$

+	[0]	[1]	[2]	[3]
[0]	[0]	[1]	[2]	[3]
[1]	[1]	[2]	[3]	[0]
[2]	[2]	[3]	[0]	[1]
[3]	[3]	[0]	[1]	[2]

·	[0]	[1]	[2]	[3]
[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]
[2]	[0]	[2]	[0]	[2]
[3]	[0]	[3]	[2]	[1]

The addition and multiplication tables for the equivalence classes of $\equiv \pmod 5$

+	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[1]	[2]	[3]	[4]
[1]	[1]	[2]	[3]	[4]	[0]
[2]	[2]	[3]	[4]	[0]	[1]
[3]	[3]	[4]	[0]	[1]	[2]
[4]	[4]	[0]	[1]	[2]	[3]

·	[0]	[1]	[2]	[3]	[4]
[0]	[0]	[0]	[0]	[0]	[0]
[1]	[0]	[1]	[2]	[3]	[4]
[2]	[0]	[2]	[4]	[1]	[3]
[3]	[0]	[3]	[1]	[4]	[2]
[4]	[0]	[4]	[3]	[2]	[1]

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