▶ **Problem DM-3.9-6** Let

$$X = \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}.$$

For $x, y \in X$, set $x\ R\ y$ (i.e., $(x, y) \in R$) if $x^2 < y^2$ or $x = y$. Show that R is a partial ordering on X. Draw a diagram of R.

Proof. For proving R is a partial ordering on X, we will show that R is reflexive, antisymmetric, and transitive.

Reflexive: For each $x \in X$, $(x, x) \in R$ by definition of R.

Antisymmetric: Suppose $(x, y) \in R$ and $x \neq y$. Then, it follows from definition that $x^2 < y^2$. Since $y \neq x$ and $y^2 \nless x^2$, this implies $(y, x) \notin R$.

Transitive: Let $x, y, z \in X$. Suppose $(x, y) \in R$ and $(y, z) \in R$. We now show that $(x, z) \in R$ according to the following four cases:

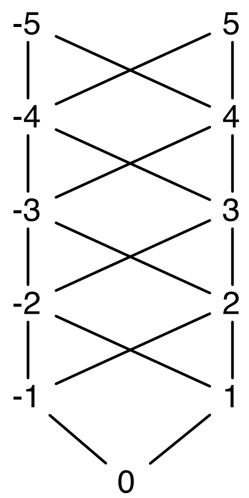
Case 1: $x = y$ and $y = z$. Then $x = z$, so $(x, z) \in R$.

Case 2: $x = y$ and $y^2 < z^2$. Then $x^2 < z^2$, so $(x, z) \in R$.

Case 3: $x^2 < y^2$ and $y = z$. Then $x^2 < z^2$, so $(x, z) \in R$.

Case 4: $x^2 < y^2$ and $y^2 < z^2$. Then $x^2 < z^2$, so $(x, z) \in R$.

Since R is reflexive, antisymmetric, and transitive, R is a partial order. In addition, we show the diagram of R as follows.



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