▶ **Problem 0.2-21** Let x be a real number. Find a necessary and sufficient condition for $x + \frac{1}{x} \geq 2$. Prove your answer.

Proof. We assert that $x + \frac{1}{x} \geq 2$ if and only if $x > 0$.

($\longrightarrow$) We offer a proof by contradiction. Suppose $x + \frac{1}{x} \geq 2$ but $x \leq 0$. If $x = 0$, then $\frac{1}{x}$ is not defined, so $x < 0$. In this case, however, $x + \frac{1}{x} < 0$, a contradiction.

($\longleftarrow$) Conversely, assume that $x > 0$. Note that $(x-1)^2 \geq 0$ implies $x^2 - 2x + 1 \geq 0$, and thus $x^2 + 1 \geq 2x$. Division by the positive number x gives $x + \frac{1}{x} \geq 2$ as required. □