▶ **Problem 0.2-23** Prove that if n is an odd integer, there is an integer m such that $n = 8m + 1$ or $n = 8m + 3$ or $n = 8m + 5$ or $n = 8m + 7$.

Proof. Since n is an odd integer, from Exercise 22 we know that there exists an integer k such that $n = 4k + 1$ or $n = 4k + 3$. We now consider the following cases.

Case 1: $n = 4k + 1$. If k is even, there exists an integer m such that $k = 2m$, so $n = 4(2m) + 1 = 8m + 1$. On the other hand, if k is odd, there exists an integer m such that $k = 2m + 1$, so $n = 4(2m + 1) + 1 = 8m + 5$. Thus, the desired conclusion is true.

Case 2: $n = 4k + 3$. If k is even, there exists an integer m such that $k = 2m$, so $n = 4(2m) + 3 = 8m + 3$. On the other hand, if k is odd, there exists an integer m such that $k = 2m + 1$, so $n = 4(2m + 1) + 3 = 8m + 7$. Thus, the desired conclusion is true. □