► **Problem 0.2-36** Suppose $f(n)$ is a polynomial such that $f(n)$ is a prime number for all $n \geq 0$. Show that $f(n) = p$ for some prime p.

Proof. We suppose the the statement "$f(n) = p$ for some prime p" is false (i.e., f is not a constant function for prime p) and let $f(n) = a_0 + a_1 n + a_2 n^2 + \cdots + a_t n^t$ for some $t \geq 1$. Since $f(n)$ is prime for all $n \geq 0$, $f(0) = a_0$ is a prime. Clearly, $f(n) = a_0 + ng(n)$ where $g(n) = a_1 + a_2 n + \cdots + a_t n^{t-1}$. Thus, $f(a_0 n) = a_0 + a_0 n g(a_0 n)$ is a prime. However, the right hand side is divisible by the prime a_0. This means that $g(a_0 n) = 0$, contradicting the fact that polynomial has only finitely many roots. □