

► **Problem 0.2-38** Prove that there exist irrational numbers a and b such that a^b is rational.

Proof. We have proven in the textbook that $\sqrt{2}$ is irrational. Thus, if $\sqrt{2}^{\sqrt{2}}$ is rational, we are done (with $a = b = \sqrt{2}$). On the other hand, if $\sqrt{2}^{\sqrt{2}}$ is irrational, then let $a = \sqrt{2}^{\sqrt{2}}$ and $b = \sqrt{2}$. In this case, $a^b = (\sqrt{2}^{\sqrt{2}})^{\sqrt{2}} = \sqrt{2}^2 = 2$ is rational. \square