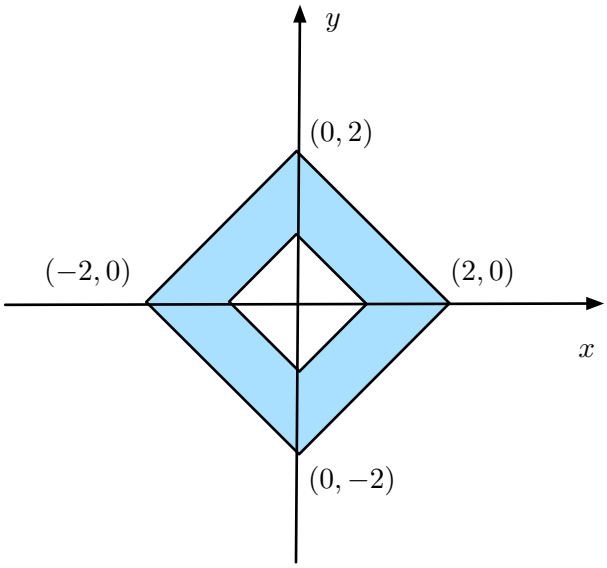
▶ **Problem 2.3-10** Define $\mathcal{R}$ on $\mathbf{R}$ by $(x, y) \in \mathcal{R}$ if and only if $1 \leq |x| + |y| \leq 2$.

(a) Make a sketch in the Cartesian plane showing the region of $\mathbf{R}^2$ defined by $\mathcal{R}$.

(b) Show that $\mathcal{R}$ is neither reflexive nor transitive.

(c) Is $\mathcal{R}$ symmetric? Is it antisymmetric? Explain.

Solution. (a)



(b) The relation is not reflexive because, for example, $(0, 0) \notin \mathcal{R}$. It is not transitive because, for example, $(2, 0) \in \mathcal{R}$ and $(0, 1) \in \mathcal{R}$ but $(2, 1) \notin \mathcal{R}$.

(c) The relation is symmetric since if $(x, y) \in \mathcal{R}$, then $1 \leq |x| + |y| \leq 2$, so $1 \leq |y| + |x| \leq 2$, and thus $(y, x) \in \mathcal{R}$. It is not antisymmetric since, for example, $(0, 1) \in \mathcal{R}$ and $(1, 0) \in \mathcal{R}$, but $0 \neq 1$. □