► **Problem 2.4-04** Let $A = \{1, 2, 3, 4, 5, 6\}$ and let $S = \mathcal{P}(A)$, the power set of A.

(a) For $a, b \in S$, define $a \sim b$ if a and b have the same number of elements. Prove that $\sim$ defines an equivalence relation on S.

(b) How many equivalence classes are there? List one element from each equivalence class.

Proof. (a) To show that $\sim$ is an equivalence on S, we verify the reflexive, symmetric, and transitive as follows.

(i). **Reflexive** : If $a \in S$, then a and a have the same number of elements, so $a \sim a$.

(ii). **Symmetric** : For any $a, b \in S$, if $a \sim b$ then a and b have the same number of elements, so b and a have the same number of elements. Thus $b \sim a$.

(iii). **Transitive** : For any $a, b, c \in S$, if $a \sim b$ and $b \sim c$, then a and b have the same number of elements, and b and c have the same number of elements. This implies that a and c have the same number of elements. Thus, $a \sim c$.

(b) There are seven equivalence classes, representing by $\emptyset$, $\{1\}$, $\{1, 2\}$, $\{1, 2, 3\}$, $\{1, 2, 3, 4\}$, $\{1, 2, 3, 4, 5\}$, $\{1, 2, 3, 4, 5, 6\}$. □