

► **Problem 4.2-12(g)(j)** In each of the following cases, find the greatest common divisor of a and b and express it in the form $ma + nb$ for suitable integers m and n .

(g) $a = -3719$ and $b = 8416$.

(j) $a = 12345$ and $b = 54321$.

Solution. (g)

$$\begin{array}{r|rr} 8416 & 1 & 0 \\ 3719 & 0 & 1 \\ 978 & 1 & -2 \\ 785 & -3 & 7 \\ 193 & 4 & -9 \\ 13 & -19 & 43 \\ 11 & 270 & -611 \\ 2 & -289 & 654 \\ 1 & 1715 & -3881 \end{array}$$

The last nonzero remainder is 1, so $\gcd(8416, 3719) = 1 = (1715) \times (8416) + (-3881) \times (3719)$. Thus, $\gcd(-3719, 8416) = 1 = (3881) \times (-3719) + (1715) \times (8416)$.

(j)

$$\begin{array}{r|rr} 54321 & 1 & 0 \\ 12345 & 0 & 1 \\ 4941 & 1 & -4 \\ 2463 & -2 & 9 \\ 15 & 5 & -22 \\ 3 & -822 & 3617 \end{array}$$

Since the last nonzero remainder is 3, $\gcd(12345, 54321) = 3 = (-822) \times (54321) + (3617) \times (12345)$. □