▶ **Problem 4.3-18(b)(c)**

(b) Is $2^{91} - 1$ prime? Explain your answer.

(c) Show that if $2^n - 1$ is prime then necessarily n is prime.

Solution.

(b) No, because $2^{91} - 1 = (2^{13} - 1)(2^{78} + 2^{65} + 2^{52} + 2^{39} + 2^{26} + 2^{13} + 1)$.

(c) If n is not a prime, we write $n = rs$ for $1 < r, s < n$. Then $2^n - 1 = 2^{rs} - 1 = (2^r - 1)(2^{r(s-1)} + 2^{r(s-2)} + \cdots + 2^r + 1)$ is not a prime, a contradiction. $\square$