▶ **Problem 4.3-32(b)(c)** Let a and b be integers. Let p be a prime. Answer true or false and explain:
(b) If $p|a$ and $p|(a^2 + b^2)$, then $p|b$.
(c) If $p|(a^9 + a^{17})$, then $p|a$.

Solution. (b) True. Since $p|a$, it implies $p|a^2$. Now, $p|a^2$ and $p|(a^2 + b^2)$ forces $p|b^2$. Then, by Proposition 4.3.7, we conclude that $p|b$.

(c) False. Consider $p = 257$ and $a = 2$. Clearly, p is a prime. Then $2^9 + 2^{17} = 131584$ and $257|(2^9 + 2^{17})$. However, $257 \nmid 2$. □