► **Problem 4.3-35**

Suppose $p$ and $p+2$ are twin primes and $p > 3$. Prove that $6|(p+1)$.

**Proof.** Since $p$ and $p+2$ are primes and $p > 3$, it implies that $p+1$ is an even integer, i.e., $2|(p+1)$. Also, we have known that there is a multiple of 3 in any three consecutive positive integers $p$, $p+1$ and $p+2$. Again, by the fact that $p$ and $p+2$ are primes, it must be $3|(p+1)$. Therefore, we have $6|(p+1)$. □

**Another Proof.** Write $p+1 = 6q + r$ with $0 \leqslant r < 6$ for some integer $q$. Consider the following cases.

If $r = 1$, then $p = 6q$, contradicting that $p$ is a prime.

If $r = 2$, then $p = 6q + 1$, so $p + 2 = 6q + 3$. Thus, $3|(p+2)$, contradicting that $p+2$ is a prime.

If $r = 3$, then $p = 6q + 2$ is divisible by 2, contradicting that $p$ is a prime.

If $r = 4$, then $p = 6q + 3$ is divisible by 3, contradicting that $p$ is a prime.

If $r = 5$, then $p = 6q + 4$ is divisible by 2, contradicting that $p$ is a prime.

Therefore, the only possibility is that $r = 0$, that is, $6|(p+1)$. □