▶ **Problem 4.4-9(i)(j)** Find all integers x, $0 \leqslant x < n$, satisfying each of the following congruences mod n. If no such x exists, explain why not.
(b) $65x \equiv 27 \pmod{n}$, $n = 169$.
(c) $4x \equiv 320 \pmod{n}$, $n = 592$.

Solution. (i) No x exists. If $65x \equiv 27 \pmod{169}$, then $169|(65x - 27)$, so $13|(65x - 27)$. Since $13|65x$, it implies $13|27$, a contradiction.

(j) If $4x \equiv 320 \pmod{592}$, then $4x = 320+592k$ for some $k \in \mathbf{Z}$. That is, $x = 80+148k$. The value of x are 80, 288, 376, 524. □