▶ **Problem 4.4-11(d)**

Find all integers x and y, $0 \leqslant x, y < n$, that satisfy each of the following pair of congruences. If no x, y exist, explain why not.

$$\begin{cases} 7x + 2y \equiv 3 \pmod{n} & n = 15 \\ 9x + 4y \equiv 6 \pmod{n} \end{cases}$$

Solution. Multiply the first congruence by 2 to get $14x + 4y \equiv 6 \pmod{15}$. Subtracting the second congruence gives $5x \equiv 0 \pmod{15}$, so $x = 0, 3, 6, 9$ or 12.

If $x = 0$, then $2y \equiv 3 \pmod{15}$, and so $y = 9$.

If $x = 3$, then $2y \equiv 3 - 21 = -18 \equiv 12 \pmod{15}$, and so $y = 6$.

If $x = 6$, then $2y \equiv 3 - 42 = -39 \equiv 6 \pmod{15}$, and so $y = 3$.

If $x = 9$, then $2y \equiv 3 - 63 = -60 \equiv 0 \pmod{15}$, and so $y = 0$.

If $x = 12$, then $2y \equiv 3 - 84 = -81 \equiv 9 \pmod{15}$, and so $y = 12$. $\square$