▶ **Problem 4.4-11(e)**

Find all integers x and y, $0 \leqslant x, y < n$, that satisfy each of the following pair of congruences. If no x, y exist, explain why not.

$$\begin{cases} 3x + 5y \equiv 14 \pmod{n} & n = 28 \\ 5x + 9y \equiv 6 \pmod{n} \end{cases}$$

Solution. Multiply the first congruence by 5 and the second by 3 gives

$$\begin{cases} 15x + 25y \equiv 70 \equiv 14 \pmod{28} \\ 15x + 27y \equiv 18 \pmod{28}. \end{cases}$$

Subtracting the first congruence from the second gives $2y \equiv 4 \pmod{28}$. Thus, $y = 2$ or $y = 16$.

If $y = 2$, then $3x \equiv 14 - 10 = 4 \pmod{28}$, and so $x = 20$.

If $y = 16$, then $3x \equiv 14 - 80 = -66 \equiv 18 \pmod{28}$, and so $x = 6$. □