► **Problem 4.4-16** Prove that an integer $(a_{n-1}a_{n-2}\cdots a_0)_{10}$ is divisible by 11 if and only if $a_0 + a_2 + a_4 + \cdots \equiv a_1 + a_3 + a_5 + \cdots \pmod{11}$. [Hint: $10 \equiv -1 \pmod{11}$.]

Proof. From the hint $10 \equiv -1 \pmod{11}$ and Proposition 4.4.7, we note that

$$10^k \equiv (-1)^k \pmod{11}$$

for any natural number k. Since

$$(a_{n-1}a_{n-2}\cdots a_2a_1a_0)_{10} = a_{n-1}\cdot 10^{n-1} + a_{n-2}\cdot 10^{n-2} + \cdots + a_3\cdot 10^3 + a_2\cdot 10^2 + a_1\cdot 10 + a_0$$

we have

$$\begin{aligned} & (a_{n-1}a_{n-2}\cdots a_2a_1a_0)_{10} \\ \equiv\ & a_{n-1}\cdot(-1)^{n-1} + a_{n-2}\cdot(-1)^{n-2} + \cdots - a_3 + a_2 - a_1 + a_0 \pmod{11}. \end{aligned}$$

and this is 0 (mod 11) if and only if $a_0 + a_2 + a_4 + \cdots \equiv a_1 + a_3 + a_5 + \cdots \pmod{11}$. □