▶ **Problem 4.5-15** Does the test in (7) detect transposition errors: that is, will it notice if two (different) adjacent digits have ben interchange? Explain.

The check digit for Universal Product Codes is determined by the rule:

(7) 3(sum of digits in odd positions)+(sum of digits in even positions)≡ 0 (mod 10).

Solution. Suppose that a_1–$a_2a_3a_4a_5a_6$–$a_7a_8a_9a_{10}a_{11}$–x is a valid universal product code. Thus,

$$A = 3(a_1 + a_3 + a_5 + a_7 + a_9 + a_{11}) + (a_2 + a_4 + a_6 + a_8 + a_{10} + x) \equiv 0 \ (\text{mod } 10)$$

If a_2 and a_3 are transposed, the error will not be detected provided

$$B = 3(a_1 + a_2 + a_5 + a_7 + a_9 + a_{11}) + (a_3 + a_4 + a_6 + a_8 + a_{10} + x) \equiv 0 \ (\text{mod } 10)$$

Now $A - B = 3(a_3 - a_2) + (a_2 - a_3) = 2(a_3 - a_2)$. So, if $a_3 - a_2 = 5$, we'll have $A - B \equiv 0 \ (\text{mod } 10)$. Hence, $B \equiv A \equiv 0 \ (\text{mod } 10)$ and the transposition will not be detected. We conclude that the test given by (7) does **not** detect errors due to the transposition of adjacent digits. □