▶ **Problem 4.5-25(b)**

Suppose $p = 17$, $q = 59$, and $s = 3$. If you receive $E = 926$, what is the message?

Solution. We first note that $\gcd(3, 16) = 1$ and $11(3) + (-2)(16) = 1$, so $a = 11$. Also, $\gcd(3, 58) = 1$ and $39(3) + (-2)(58) = 1$, so $b = 39$. Then $E^a = 926^{11} \equiv 8^{11} = 8(8^2)^5 \equiv 8 \cdot 13^5 \equiv 2 \pmod{17}$, while $E^b = 926^{39} \equiv 41^{39} = 41(41^2)^{19} \equiv 41 \cdot 29^{19} = 41(29^3)(29^4)^4 \equiv 41 \cdot 22 \cdot 48^4 \equiv 41 \cdot 22 \cdot 3^2 = 8118 \equiv 35 \pmod{59}$. Thus,

$$E^a \equiv 2 \pmod{17}$$

$$E^a \equiv 35 \pmod{59}$$

Since $1 = 7(17) - 2(59)$, we have $E = 2 \cdot (-2) \cdot 59 + 35 \cdot 7 \cdot 17 = 3929 \equiv 920 \pmod{1003}$. The message is **IT**. □