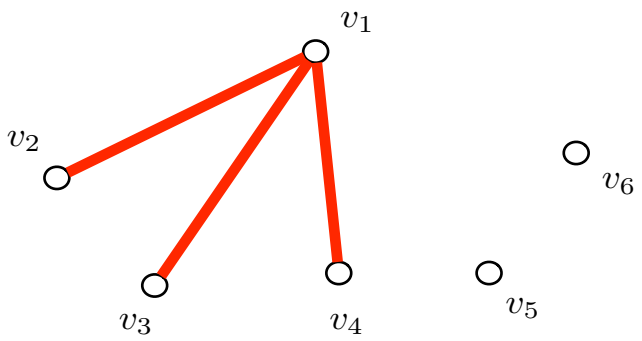
▶ **Problem 9.1-08**

(a) A graph has six vertices, every two of which are joined by an edge. Each edge is colored red or white. Show that the graph contains a monochromatic triangle (a triangle all of whose vertices have the same color.)

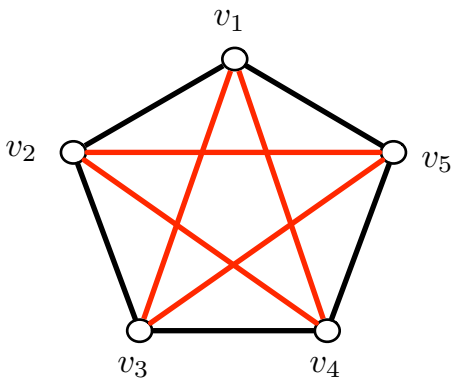
(b) Is the result of (a) true for a graph with five vertices? Explain.

Proof. (a) Suppose that the six vertices are labeled by $v_1, v_2, \ldots, v_6$. Consider v_1 in the graph. Since v_1 is incident with five edges, it follows by the Pigeonhole Principle that at least three of these five edges are colored the same color, say red. Suppose that v_1v_2, v_1v_3 and v_1v_4 are three red edges, as shown in the following Figure.



If any of the edges v_2v_3, v_2v_4 and v_3v_4 is colored by red, then we have a red triangle; otherwise, all of these three edges are colored by white, and a white triangle is formed.

(b) As shown in the following figure, the result of (a) is not true for a graph with five vertices.



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