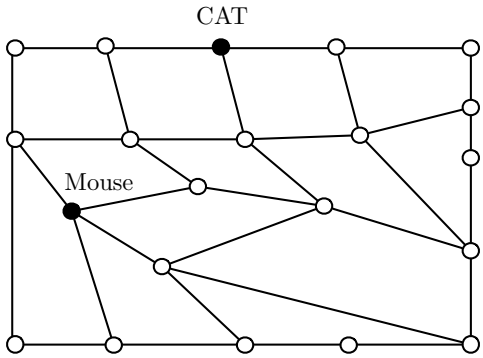
▶ **Problem 9.1-11**

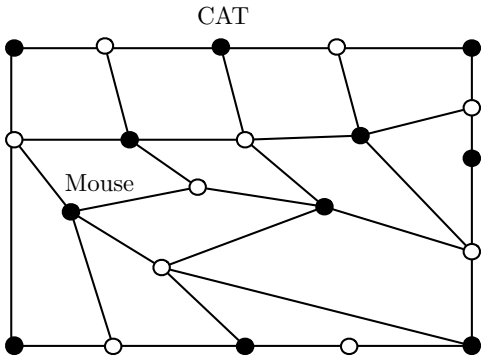
A cat and mouse are at the position indicated in a maze depicted by the graph shown below.



(a) Color the cat's and the mouse's positions black. Then color the remaining vertices black and white in such a way that every path in the maze connects vertices of different colors.

(b) The cat would like to catch the mouse, of course, so the cat change position to a white vertices. Then the mouse moves, then the cat, and so on, each animal moving alternately one after the other and always to a vertex of the other color. Show that the cat will never get her mouse.

Solution. (a) Such a coloring is shown below.



Proof. (b) Suppose the cat and the mouse are each at black positions, as they are initially, and it is the cat's move. The cat must move to a white position, so she doesn't catch the mouse since the mouse is at a black position.

Since every vertex in the graph is joined to at least two other vertices, the mouse can always move to a vertex where the cat isn't in there. Now both animals are at white vertices and it's the cat's turn to move. The cat moves to a black vertex where the mouse isn't in there (the mouse is at a white position). Again, the mouse can move to a (necessarily black) position different from the cat's. And so on. □