▶ Problem 9.2-31

A graph G with 17 edges has the property that all its vertices have the same degree d. Find all possible values for d. For each such value, state briefly what the corresponding graph looks like.

Solution. Suppose G has vertex set V and edge set E. Since $\sum_{v \in V} \deg v = 2|E|$, we have $d \cdot n = 34$, so d = 1, 2, 17, 34, respectively. Clearly, the cases d = 34, n = 1 and d = 17, n = 2 are impossible. The case d = 1, n = 34 defines a graph with 17 components each of which is a single edge. The case d = 2, n = 17 defines a cycle containing 17 vertices.