▶ **Review 0-15** Prove that $\sqrt{3}$ is not a rational number.

Proof. We suppose that $\sqrt{3} = \frac{a}{b}$ is a rational number, where a and b are integers. Without loss of generality, we may assume that a and b have no factors in common. Squaring both sides of $\sqrt{3} = \frac{a}{b}$ gives $a^2 = 3b^2$, which is a multiple of 3. Recall that if $a = 3k + 1$ then $a^2 = (3k + 1)^2 = 3(3k^2 + 2k) + 1$ is not a multiple of 3. Similarly, if $a = 3k + 2$ then $a^2 = (3k + 2)^2 = 3(3k^2 + 4k + 1) + 1$ is not a multiple of 3. Since a^2 is a multiple of 3, it implies that $a = 3k$ is a multiple of 3. Thus, $3b^2 = 9k^2$ and $b^2 = 3k^2$ is a multiple of 3. This again implies that b is also a multiple of 3, contradicting our assumption that a and b have no factor in common. □