

► **Review 1-06** Establish the logical equivalence of each of the following pairs of statements.

(a)  $[(p \rightarrow q) \rightarrow r]$  and  $[(p \vee r) \wedge (\neg(q \wedge (\neg r)))]$ .

(b)  $p \rightarrow (q \vee r)$  and  $[p \wedge (\neg q)] \rightarrow r$ .

**Solution.** (a)

$$\begin{aligned} (p \rightarrow q) \rightarrow r &\Leftrightarrow ((\neg p) \vee q) \rightarrow r \\ &\Leftrightarrow \neg((\neg p) \vee q) \vee r \\ &\Leftrightarrow (p \wedge (\neg q)) \vee r \\ &\Leftrightarrow (p \vee r) \wedge ((\neg q) \vee r) \\ &\Leftrightarrow (p \vee r) \wedge (\neg(q \wedge (\neg r))) \end{aligned}$$

(b)

$$\begin{aligned} p \rightarrow (q \vee r) &\Leftrightarrow (\neg p) \vee (q \vee r) \\ &\Leftrightarrow ((\neg p) \vee q) \vee r \\ &\Leftrightarrow \neg(p \wedge (\neg q)) \vee r \\ &\Leftrightarrow (p \wedge (\neg q)) \rightarrow r \end{aligned}$$

□