

► **Review 1-07** Express each of the following statements in disjunctive normal form.

$$(a) ((p \vee q) \wedge r) \vee ((p \vee q) \wedge (\neg p)).$$

$$(b) [p \vee (q \wedge (\neg r))] \wedge \neg(q \wedge r).$$

Solution. (a)

$$\begin{aligned} & ((p \vee q) \wedge r) \vee ((p \vee q) \wedge (\neg p)) \\ \Leftrightarrow & (p \wedge r) \vee (q \wedge r) \vee (p \wedge (\neg p)) \vee (q \wedge (\neg p)) \\ \Leftrightarrow & (p \wedge r) \vee (q \wedge r) \vee \mathbf{0} \vee ((\neg p) \wedge q) \\ \Leftrightarrow & (p \wedge r) \vee (q \wedge r) \vee ((\neg p) \wedge q) \\ \Leftrightarrow & [(p \wedge q \wedge r) \vee (p \wedge (\neg q) \wedge r)] \vee [(p \wedge q \wedge r) \vee ((\neg p) \wedge q \wedge r)] \vee \\ & [((\neg p) \wedge q \wedge r) \vee ((\neg p) \wedge q \wedge (\neg r))] \\ \Leftrightarrow & (p \wedge q \wedge r) \vee (p \wedge (\neg q) \wedge r) \vee ((\neg p) \wedge q \wedge r) \vee ((\neg p) \wedge q \wedge (\neg r)) \end{aligned}$$

(b)

$$\begin{aligned} & (p \vee (q \wedge (\neg r))) \wedge \neg(q \wedge r) \\ \Leftrightarrow & (p \vee (q \wedge (\neg r))) \wedge ((\neg q) \vee (\neg r)) \\ \Leftrightarrow & ((p \vee (q \wedge (\neg r))) \wedge (\neg q)) \vee ((p \vee (q \wedge (\neg r))) \wedge (\neg r)) \\ \Leftrightarrow & (p \wedge (\neg q)) \vee (q \wedge (\neg r) \wedge (\neg q)) \vee (p \wedge (\neg r)) \vee (q \wedge (\neg r) \wedge (\neg r)) \\ \Leftrightarrow & [(p \wedge (\neg q) \wedge r) \vee (p \wedge (\neg q) \wedge (\neg r))] \vee \mathbf{0} \vee [(p \wedge q \wedge (\neg r)) \vee (p \wedge (\neg q) \wedge (\neg r))] \vee \\ & [(p \wedge q \wedge (\neg r)) \vee ((\neg p) \wedge q \wedge (\neg r))] \\ \Leftrightarrow & (p \wedge (\neg q) \wedge r) \vee (p \wedge (\neg q) \wedge (\neg r)) \vee (p \wedge q \wedge (\neg r)) \vee ((\neg p) \wedge q \wedge (\neg r)) \end{aligned}$$

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