▶ **Review 1-08** Determine whether each of the following arguments is valid.

(a)

$$\frac{\begin{array}{c} p \to q \\ \neg p \end{array}}{\neg q}$$

(b)

$$\frac{\begin{array}{c} \neg((\neg p) \wedge q) \\ \neg(p \wedge r) \\ r \vee s \end{array}}{q \to s}$$

(c)

$$\frac{\begin{array}{c} p \vee (\neg q) \\ (t \vee s) \to (p \vee r) \\ (\neg r) \vee (t \vee s) \\ p \leftrightarrow (t \vee s) \end{array}}{(p \wedge r) \to (q \wedge r)}$$

Solution. (a) This argument is not valid because if p is false and q is true, then the premises are true but the conclusion is not.

(b) The first hypothesis can be rewritten as $p \vee (\neg q)$, which is the same as $q \to p$. The second hypothesis is $(\neg p) \vee (\neg r)$, which is the same as $p \to (\neg r)$. The third hypothesis is $(\neg r) \to s$. So the given argument is

$$\frac{\begin{array}{c} q \to p \\ p \to (\neg r) \\ (\neg r) \to s \end{array}}{q \to s}$$

Using the chain rule twice tell us this is valid.

(c) This argument is not valid because if p, r and t are true while q is false (and s takes on either true or false), then the hypotheses are true while the conclusion is false. □