▶ **Problem 10.1-17**

Let u and v be distinct vertices in a graph G. Prove that there is a walk from u to v if and only if there is a path from u to v.

Proof. By definition, every path in a graph is a walk. Thus, we only need to prove that the existence of a walk from u to v implies the existence of a path from u to v. Suppose that

$$u = u_0, u_1, \ldots, u_k = v$$

is a walk from u to v. If there are no repeated vertices then this walk is already a path. Suppose that some vertex is repeated in the walk. That is, there exist two indices i and j with $i < j$ such that $u_i = u_j$. Then, we delete the vertices $u_{i+1}, u_{i+2}, \ldots, u_j$ from the walk to obatin another walk

$$u = u_0, u_1, \ldots, u_i, u_{j+1}, \ldots, u_k = v$$

with fewer vertices than the first walk. If it contains no repeated vertices, it's the desired path; otherwise, repeating the above procedure, we obtain a shorter walk. This process continues until we get a walk without repeated vertices, in other words, a path. □