## - Problem 10.2-6

(a) (The Knight's Tour) Is it possible for a knight to tour a chessboard visiting every square exactly once and returning to its initial square?
(b) Is the sort of tour described in (a) possible on a $7 \times 7$ "chessboard"?

Solution. (a) Yes, it is possible. Here is a Hamiltonian cycle for the knight.

| 5 | 64 | 25 | 28 | 41 | 44 | 53 | 64 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 24 | 27 | 4 | 63 | 52 | 47 | 40 | 43 |
| 1 | 6 | 17 | 26 | 29 | 42 | 45 | 54 |
| 18 | 23 | 62 | 3 | 48 | 51 | 34 | 39 |
| 7 | 2 | 19 | 16 | 35 | 30 | 55 | 50 |
| 22 | 13 | 10 | 61 | 58 | 49 | 38 | 33 |
| 11 | 8 | 15 | 20 | 31 | 36 | 59 | 56 |
| 14 | 21 | 12 | 9 | 60 | 57 | 32 | 37 |

(b) Such a tour is impossible because a $7 \times 7$ board has an odd number of square. A knight's tour corresponds to an alternating set of distinct red and black squares. Since we require that the knight return to the square from which he left, the number of squares of the tour must be even.

