▶ **Problem 10.2-6**

(a) How many Hamiltonian cycles does K_n have?

(b) Find all the Hamiltonian cycles in K_n for $n = 1, 2, 3, 4, 5$. In each case, exhibit a maximum number that are edge disjoint.

Solution. (a) If $n = 2$, there are no cycles, so we consider $n \geqslant 3$. Suppose that the vertices of K_n are $1, 2, \ldots, n$. If we begin at vertex 1, there are $n-1$ choices for the second vertex in a cycle, then $n-2$ choices for the third vertex and so on. Since a Hamiltonian cycle is an undirected cycle, there are $\frac{1}{2}(n-1)!$ different Hamiltonian cycles in K_n.

(d) If $n = 2$, there are no Hamiltonian cycles (and therefore no edge disjoint ones).

If $n = 3$, then 1231 the only Hamiltonian cycle; so there are no edge disjoint Hamiltonian cycles.

If $n = 4$, the Hamiltonian cycles are 12341, 12431 and 13241. No pair are edge disjoint.

If $n = 5$, the Hamiltonian cycles are 123451, 123541, 124351, 124531, 125341, 125431, 132451, 132541, 134251, 135241, 142351, 143251. Since $\frac{n-1}{2} = 2$, we could possible have two edge disjoint cycles. For example, 123451 and 135241. □