▶ Problem 10.3-05

Let A be the adjacency matrix of a graph G whose vertex set is $\{v_1, \ldots, v_n\}$. Prove that the *i*th entry on the diagonal of A^3 equals twice the number of different triangles that contain vertex v_i .

Proof. The *i*th entry on the diagonal of A^3 is the number of walks of length 3 from v_i to itself. However, a closed walk of length 3 in a graph must give a triangle. Every triangle can be walked in exactly two different directions. Hence, the number of walks of length 3 is twice the number of triangles.