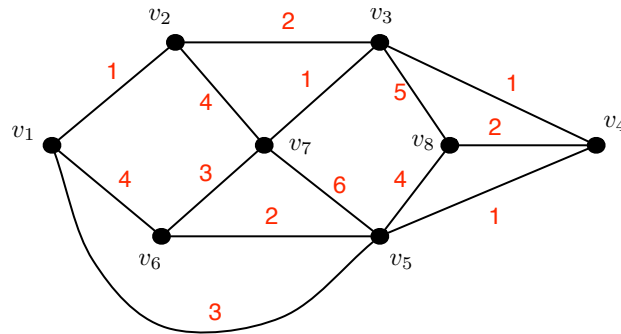


► **Problem 10.4-18**

The Floyd-Warshall algorithm is applied to the graph shown.

- (a) Find the final values of $d(7, 1), d(7, 2), \dots, d(7, 8)$.
- (b) Find the values of $d(1, 2), d(3, 4), d(2, 5)$, and $d(8, 6)$ after $k = 4$.
- (c) Find the values of $d(6, 8)$ at the start and as k varies from 1 to 8.



Solution.

| 初始值 | | | | | | | | |
|-----|----|----|----|----|----|----|----|----|
| | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 |
| V1 | 0 | 1 | ∞ | ∞ | 3 | 4 | ∞ | ∞ |
| V2 | 1 | 0 | 2 | ∞ | ∞ | ∞ | 4 | ∞ |
| V3 | ∞ | 2 | 0 | 1 | ∞ | ∞ | 1 | 5 |
| V4 | ∞ | ∞ | 1 | 0 | 1 | ∞ | ∞ | 2 |
| V5 | 3 | ∞ | ∞ | 1 | 0 | 2 | 6 | 4 |
| V6 | 4 | ∞ | ∞ | ∞ | 2 | 0 | 3 | ∞ |
| V7 | ∞ | 4 | 1 | ∞ | 6 | 3 | 0 | ∞ |
| V8 | ∞ | ∞ | 5 | 2 | 4 | ∞ | ∞ | 0 |

| After k=1 | | | | | | | | |
|-----------|----|----|----|----|----|----|----|----|
| | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 |
| V1 | 0 | 1 | ∞ | ∞ | 3 | 4 | ∞ | ∞ |
| V2 | 1 | 0 | 2 | ∞ | 4 | 5 | 4 | ∞ |
| V3 | ∞ | 2 | 0 | 1 | ∞ | ∞ | 1 | 5 |
| V4 | ∞ | ∞ | 1 | 0 | 1 | ∞ | ∞ | 2 |
| V5 | 3 | 4 | ∞ | 1 | 0 | 2 | 6 | 4 |
| V6 | 4 | 5 | ∞ | ∞ | 2 | 0 | 3 | ∞ |
| V7 | ∞ | 4 | 1 | ∞ | 6 | 3 | 0 | ∞ |
| V8 | ∞ | ∞ | 5 | 2 | 4 | ∞ | ∞ | 0 |

| After k=2 | | | | | | | | |
|-----------|----|----|----|----|----|----|----|----|
| | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 |
| V1 | 0 | 1 | 3 | ∞ | 3 | 4 | 5 | ∞ |
| V2 | 1 | 0 | 2 | ∞ | 4 | 5 | 4 | ∞ |
| V3 | 3 | 2 | 0 | 1 | 6 | 7 | 1 | 5 |
| V4 | ∞ | ∞ | 1 | 0 | 1 | ∞ | ∞ | 2 |
| V5 | 3 | 4 | 6 | 1 | 0 | 2 | 6 | 4 |
| V6 | 4 | 5 | 7 | ∞ | 2 | 0 | 3 | ∞ |
| V7 | 5 | 4 | 1 | ∞ | 6 | 3 | 0 | ∞ |
| V8 | ∞ | ∞ | 5 | 2 | 4 | ∞ | ∞ | 0 |

| After k=3 | | | | | | | | |
|-----------|----|----|----|----|----|----|----|----|
| | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 |
| V1 | 0 | 1 | 3 | 4 | 3 | 4 | 4 | 8 |
| V2 | 1 | 0 | 2 | 3 | 4 | 5 | 3 | 7 |
| V3 | 3 | 2 | 0 | 1 | 6 | 7 | 1 | 5 |
| V4 | 4 | 3 | 1 | 0 | 1 | 8 | 2 | 2 |
| V5 | 3 | 4 | 6 | 1 | 0 | 2 | 6 | 4 |
| V6 | 4 | 5 | 7 | 8 | 2 | 0 | 3 | 12 |
| V7 | 4 | 3 | 1 | 2 | 6 | 3 | 0 | 6 |
| V8 | 8 | 7 | 5 | 2 | 4 | 12 | 6 | 0 |

| After k=4 | | | | | | | | |
|-----------|----|----|----|----|----|----|----|----|
| | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 |
| V1 | 0 | 1 | 3 | 4 | 3 | 4 | 4 | 6 |
| V2 | 1 | 0 | 2 | 3 | 4 | 5 | 3 | 5 |
| V3 | 3 | 2 | 0 | 1 | 2 | 7 | 1 | 3 |
| V4 | 4 | 3 | 1 | 0 | 1 | 8 | 2 | 2 |
| V5 | 3 | 4 | 2 | 1 | 0 | 2 | 6 | 3 |
| V6 | 4 | 5 | 7 | 8 | 2 | 0 | 3 | 10 |
| V7 | 4 | 3 | 1 | 2 | 6 | 3 | 0 | 4 |
| V8 | 6 | 5 | 3 | 2 | 3 | 10 | 4 | 0 |

| After k=5 | | | | | | | | |
|-----------|----|----|----|----|----|----|----|----|
| | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 |
| V1 | 0 | 1 | 3 | 4 | 3 | 4 | 4 | 6 |
| V2 | 1 | 0 | 2 | 3 | 4 | 5 | 3 | 5 |
| V3 | 3 | 2 | 0 | 1 | 2 | 4 | 1 | 3 |
| V4 | 4 | 3 | 1 | 0 | 1 | 3 | 2 | 2 |
| V5 | 3 | 4 | 2 | 1 | 0 | 2 | 3 | 3 |
| V6 | 4 | 5 | 4 | 3 | 2 | 0 | 3 | 5 |
| V7 | 4 | 3 | 1 | 2 | 3 | 3 | 0 | 4 |
| V8 | 6 | 5 | 3 | 2 | 3 | 5 | 4 | 0 |

| After k=6 | | | | | | | | |
|-----------|----|----|----|----|----|----|----|----|
| | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 |
| V1 | 0 | 1 | 3 | 4 | 3 | 4 | 4 | 6 |
| V2 | 1 | 0 | 2 | 3 | 4 | 5 | 3 | 5 |
| V3 | 3 | 2 | 0 | 1 | 2 | 4 | 1 | 3 |
| V4 | 4 | 3 | 1 | 0 | 1 | 3 | 2 | 2 |
| V5 | 3 | 4 | 2 | 1 | 0 | 2 | 3 | 3 |
| V6 | 4 | 5 | 4 | 3 | 2 | 0 | 3 | 5 |
| V7 | 4 | 3 | 1 | 2 | 3 | 3 | 0 | 4 |
| V8 | 6 | 5 | 3 | 2 | 3 | 5 | 4 | 0 |

| After k=7 | | | | | | | | |
|-----------|----|----|----|----|----|----|----|----|
| | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 |
| V1 | 0 | 1 | 3 | 4 | 3 | 4 | 4 | 6 |
| V2 | 1 | 0 | 2 | 3 | 4 | 5 | 3 | 5 |
| V3 | 3 | 2 | 0 | 1 | 2 | 4 | 1 | 3 |
| V4 | 4 | 3 | 1 | 0 | 1 | 3 | 2 | 2 |
| V5 | 3 | 4 | 2 | 1 | 0 | 2 | 3 | 3 |
| V6 | 4 | 5 | 4 | 3 | 2 | 0 | 3 | 5 |
| V7 | 4 | 3 | 1 | 2 | 3 | 3 | 0 | 4 |
| V8 | 6 | 5 | 3 | 2 | 3 | 5 | 4 | 0 |

| After k=8 | | | | | | | | |
|-----------|----|----|----|----|----|----|----|----|
| | V1 | V2 | V3 | V4 | V5 | V6 | V7 | V8 |
| V1 | 0 | 1 | 3 | 4 | 3 | 4 | 4 | 6 |
| V2 | 1 | 0 | 2 | 3 | 4 | 5 | 3 | 5 |
| V3 | 3 | 2 | 0 | 1 | 2 | 4 | 1 | 3 |
| V4 | 4 | 3 | 1 | 0 | 1 | 3 | 2 | 2 |
| V5 | 3 | 4 | 2 | 1 | 0 | 2 | 3 | 3 |
| V6 | 4 | 5 | 4 | 3 | 2 | 0 | 3 | 5 |
| V7 | 4 | 3 | 1 | 2 | 3 | 3 | 0 | 4 |
| V8 | 6 | 5 | 3 | 2 | 3 | 5 | 4 | 0 |

(a) $d(7, 1) = 4$, $d(7, 2) = 3$, $d(7, 3) = 1$, $d(7, 4) = 2$, $d(7, 5) = 3$, $d(7, 6) = 3$, $d(7, 7) = 0$, and $d(7, 8) = 4$.

(b) $d(1, 2) = 1$, $d(3, 4) = 1$, $d(2, 5) = 4$, $d(8, 6) = 10$.

(c) $\infty, \infty, \infty, 12, 10, 5, 5, 5, 5$.

□