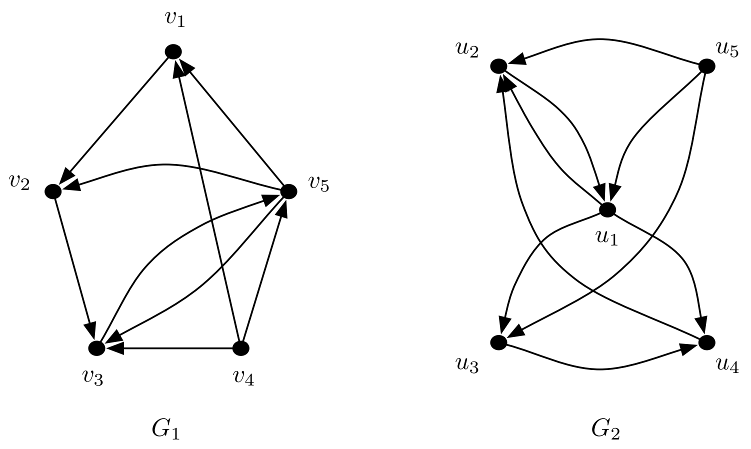
▶ Problem 11.2-15

Consider the digraphs G_1, G_2 shown.



(a) Find the adjacency matrix A_1 of G_1 and the adjacency matrix A_2 of G_2.

(b) Explain why the map $\phi : G_1 \to G_2$ defined by

$$\phi(v_1) = u_3, \quad \phi(v_2) = u_4, \quad \phi(v_3) = u_2, \quad \phi(v_4) = u_5, \quad \phi(v_5) = u_1$$

is an isomorphism.

(c) Find the permutation matrix P that corresponds to ϕ and satisfies $PA_1P^T = A_2$.

(d) Are these digraphs strongly connected?

(e) Are these digraphs Eulerian?

Solution. (a)

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

(b) With the vertices of G_1 relabeled according to ϕ, the adjacency matrix of G_1 becomes that of G_2.

(c)

$$P = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(d) The digraphs are not strongly connected. For instance, there is no path from v_5 to v_4 in G_1 and no path from u_1 to u_5 in G_2.

(e) The digraphs are not Eulerian because they are not strongly connected. □