▶ **Problem 12.1-20**

Let T be a tree with n vertices $v_1, v_2, \ldots, v_n$. Prove that the number of leaves in T is $2 + \sum\limits_{\deg v_i \geqslant 3} [\deg v_i - 2]$.

Proof. Suppose that there are k_i vertices of degree i. Since $\sum \deg v_i$ is twice the number of edges in T, we have $k_1 + 2k_2 + 3k_3 + \cdots = 2(n-1)$. Since $k_1 + k_2 + \cdots = n$,

$$k_1 + 2k_2 + 3k_3 + \cdots = 2(n-1) = 2(k_1 + k_2 + \cdots - 1) = 2k_1 + 2k_2 + 2k_3 + \cdots - 2.$$

Solving for k_1, we get

$$\begin{aligned} k_1 &= (3-2)k_3 + (4-2)k_4 + (5-2)k_5 + \cdots + 2 \\ &= \sum_{\deg v = 3} (3-2) + \sum_{\deg v = 4} (4-2) + \sum_{\deg v = 5} (5-2) + \cdots + 2 \\ &= 2 + \sum_{\deg v_i \geqslant 3} [\deg v_i - 2] \end{aligned}$$

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