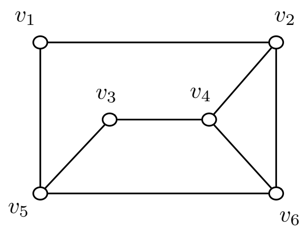
▶ **Problem 12.2-02 (c)**

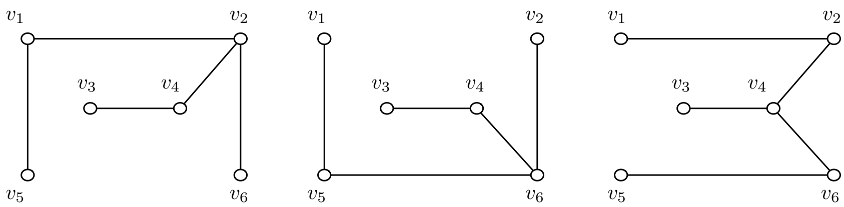
For the graph shown below,

(a) Find three spanning trees representing two isomorphism classes of graphs.

(b) Find the total number of spanning trees.



Solution. (a) Here is one possible answer. The first and second trees are isomorphic, but neither is isomorphic to the third.



(b) Kirchhoff's matrix is $\begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & 0 & -1 & 0 & -1 \\ 0 & 0 & 2 & -1 & -1 & 0 \\ 0 & -1 & -1 & 3 & 0 & -1 \\ -1 & 0 & -1 & 0 & 3 & -1 \\ 0 & -1 & 0 & -1 & -1 & 3 \end{bmatrix}$

The (6,4) cofactor is $\begin{vmatrix} 2 & -1 & 0 & -1 & 0 \\ -1 & 3 & 0 & 0 & -1 \\ 0 & 0 & 2 & -1 & 0 \\ 0 & -1 & -1 & 0 & -1 \\ -1 & 0 & -1 & 3 & -1 \end{vmatrix}$

$$= 2\begin{vmatrix} 3 & 0 & 0 & -1 \\ 0 & 2 & -1 & 0 \\ -1 & -1 & 0 & -1 \\ 0 & -1 & 3 & -1 \end{vmatrix} - (-1)\begin{vmatrix} -1 & 0 & 0 & -1 \\ 0 & 2 & -1 & 0 \\ 0 & -1 & 0 & -1 \\ -1 & -1 & 3 & -1 \end{vmatrix} - (-1)\begin{vmatrix} -1 & 3 & 0 & -1 \\ 0 & 0 & 2 & 0 \\ 0 & -1 & -1 & -1 \\ -1 & 0 & -1 & -1 \end{vmatrix}$$

$$= 2\left(3\begin{vmatrix} 2 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 3 & -1 \end{vmatrix} - (-1)\begin{vmatrix} 0 & 2 & -1 \\ -1 & -1 & 0 \\ 0 & -1 & 3 \end{vmatrix}\right) + \left((-1)\begin{vmatrix} 2 & -1 & 0 \\ -1 & 0 & -1 \\ -1 & 3 & -1 \end{vmatrix} - (-1)\begin{vmatrix} 0 & 2 & -1 \\ 0 & -1 & 0 \\ -1 & -1 & 3 \end{vmatrix}\right)$$

$$+ \left(-(2)\begin{vmatrix} -1 & 3 & -1 \\ 0 & -1 & -1 \\ -1 & 0 & -1 \end{vmatrix}\right)$$

$$= 2 \times \Big(3 \times 6 - (-1) \times (-5)\Big) + \Big((-1) \times 6 - (-1) \times 1\Big) + \Big(-2 \times 3\Big) = 35$$ □