▶ Problem 12.2-03

Let e be an edge of the complete graph K_n. Prove that the number of spanning trees of K_n that contain e is $2n^{n-3}$.

Proof. K_n has n^{n-2} spanning trees each with $n-1$ edges. Hence the total number of all edges used in all spanning trees is $(n-1)n^{n-2}$. Now each of the $\binom{n}{2}$ edges in K_n is equally likely to be included in a spanning tree. Hence, the number of spanning trees containing e is $\dfrac{(n-1)n^{n-2}}{\binom{n}{2}} = 2n^{n-3}$. $\square$