

► **Problem 13.1-18 (a)**

Show that any planar graph all of whose vertices have degree at least 5 must have at least 12 vertices.

**Proof.** If this result is true for connected planar graphs, it is true for planar graphs in general since any connected component would contain at least 12 vertices. Thus, it is sufficient to prove the result for connected graphs. Suppose  $G$  is a connected planar graph with every vertex of degree  $d \geq 5$ . Since  $\sum \deg v_i = 2E$ , then  $2E \geq 5V$ . Since  $E \leq 3V - 6$  by Theorem 13.1.4, we have  $6V - 12 \geq 2E \geq 5V$  and  $V \geq 12$ .  $\square$