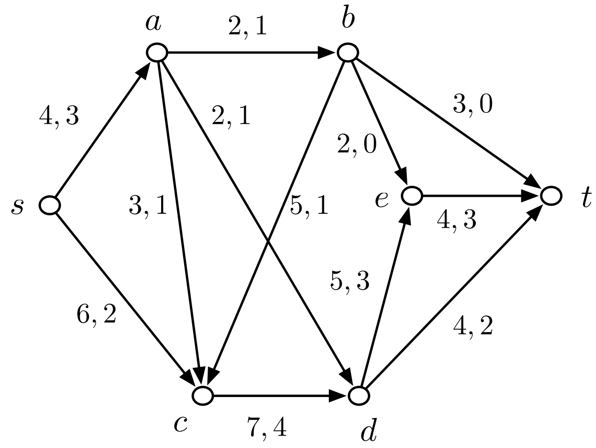
▶ **Problem 14.1-04**

For the network shown below,



(a) Verify the law of conservation of flow at a, b, and e.

(b) Find the value of the indicated flow.

(c) Find the capacity of the (s, t)-cut defined by $S = \{s, a, c, d\}$ and $T = \{b, e, t\}$.

(d) Can the flow be increased along the path $sadt$? If so, by how much? Can it be increased along $scdt$? If so, by how much?

(e) Is the given flow maximum? Explain.

(f) Illustrate Corollary 14.1.5 for S and T as in (c).

Solution. (a)

- At a, $\sum_v f_{va} = f_{sa} = 3$ and $\sum_v f_{av} = f_{ab} + f_{ac} + f_{ad} = 1 + 1 + 1 = 3$.
- At b, $\sum_v f_{vb} = f_{sb} = 1$ and $\sum_v f_{bv} = f_{bc} + f_{be} + f_{bt} = 1 + 0 + 0 = 1$.
- At e, $\sum_v f_{ve} = f_{be} + f_{de} = 0 + 3 = 3$ and $\sum_v f_{ev} = f_{et} = 3$.

(b) The value of the flow is 5.

(c) The capacity of the cut is $c_{ab} + c_{de} + c_{dt} = 2 + 5 + 4 = 11$.

(d) The flow can be increased by one unit along $sadt$ and by two units along $scdt$.

(e) The flow is not maximum, as noted in (d).

(f)
$$\begin{aligned} val(F) &= \sum_{u \in S, v \in T} (f_{uv} - f_{vu}) \\ &= (f_{ab} - f_{ba}) + (f_{cb} - f_{bc}) + (f_{de} - f_{ed}) + (f_{dt} - f_{td}) \\ &= (1 - 0) + (0 - 1) + (3 - 0) + (2 - 0) \\ &= 5 \end{aligned}$$

□