► **Problem 9.2-8**

Draw a graph with 64 vertices representing the squares of a chessboard. Two vertices are adjacent if you can move legally between the corresponding squares with a single move of a knight. [The moves of a knight are L-shaped, two squares vertically (or horizontally) followed by one square horizontally (respectively, vertically).]

(a) Explain why this graph is bipartite.

(b) What are the degrees of the vertices?

Solution. (a) We consider the chessboard as a 8×8 two-dimensional grid such that every square in the chessboard is corresponding to a vertex and two vertices are joined by an edge if and only if one can move to the other by means of a knight's move defined above. Then, the graph is shown below.
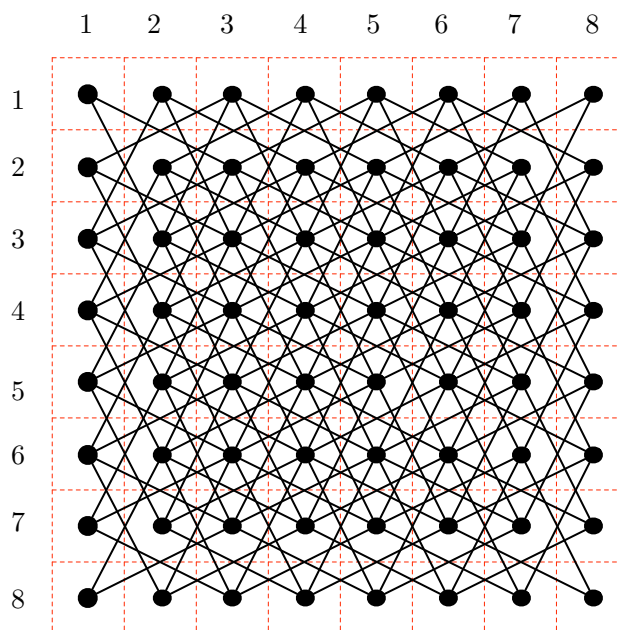
Now, if we consider that every vertex in the graph is labeled by the sum of its row number and column number. It is clear that a vertex with even label (respectively, odd label) can only be adjacent to vertices with odd label (respectively, even label). Thus, the resulting graph is indeed a bipartite graph.

(b) According to the above Figure, all degrees of vertices are shown as follows.

	1	2	3	4	5	6	7	8
1	2	3	4	4	4	4	3	2
2	3	4	6	6	6	6	4	3
3	4	6	8	8	8	8	6	4
4	4	6	8	8	8	8	6	4
5	4	6	8	8	8	8	6	4
6	4	6	8	8	8	8	6	4
7	3	4	6	6	6	6	4	3
8	2	3	4	4	4	4	3	2

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