

► **Problem 9.2-27**

Suppose a graph has nine vertices each of degree 5 or 6. Prove that at least five vertices have degree 6 or at least six vertices have degree 5.

Proof. Let G be such a graph with k vertices of degree 6 and suppose $k < 5$. Then, either $k = 4$ or $k \leq 3$. For the former case, it implies that there exist $9 - 4 = 5$ vertices of degree 5 in G , which is impossible since the number of odd vertices must be even. For the latter case, if three or fewer vertices of degree 6 in G , then it implies that G contains six or more vertices of degree 5, as required.

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