▶ **Problem 10.1-23 (b)**

Let G be a connected graph with $n > 1$ vertices. Prove that G has at least $n - 1$ edges.

Proof. We use induction on n. If $n = 2$, G must have an edge (by connectedness), so the number of edges in G is $|E| = 1 = n - 1$ and the result is true. Now assume that the result holds for connected graph with k vertices and suppose that G is connected with $k + 1$ vertices. We must show that the number of edges in G is at least k. If G has no vertex of degree one, then G has at least $k + 1$ edges by the result of the problem 10.1-23 (a), and we are done. On the other hand, we suppose that G contains a vertex w with degree one. Let H be the graph obtained from G by removing w and the edge e with which it is incident. Because a vertex of degree one cannot be an intermediate vertex of a path, H is a connected graph with k vertices. By the induction hypothesis, the smaller graph H has at least $k + 1$ edges. So G has at least $(k - 1) + 1 = k$ edges, as required. (The "+1" counts the edge e.)

□