

► **Problem 10.2-15 (d)**

- (a) [BB] Suppose G is a graph with n vertices, each of which has degree $d \geq \frac{n-1}{2}$. Prove that G contains a Hamiltonian path.
- (d) Does the converse of (a) hold; that is, if a graph has a Hamiltonian path, must the degree of every vertex be at least $\frac{n-1}{2}$? Explain your answer. what about the converse of Dirac's theorem?

Solution. (d) No. The graph P_6 (i.e., a path on 6 vertices) is a counterexample since it has a Hamiltonian path while $\frac{n-1}{2} = \frac{5}{2} = 2\frac{1}{2}$ and not all vertices have degree at least 3.

Also, the converse to Dirac's Theorem is false, as we can see that the graph C_6 (i.e., a cycle on 6 vertices) has a Hamiltonian cycle, yet not all vertices have degree at least $\frac{n}{2} = \frac{6}{2} = 3$. □