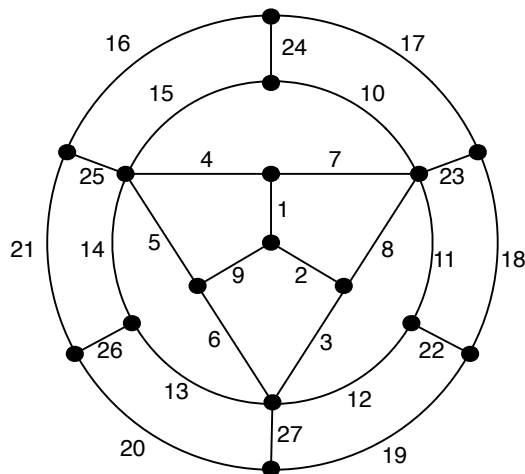


► **Problem 10.2-7**

Is the graph shown below Hamiltonian? Display a Hamiltonian cycle or explain clearly why no such cycle exists.



Solution. No, it isn't. Label edges as shown. Two of the three middle edges 1, 2, 9 must be in any Hamiltonian cycle. By symmetry, we may assume that these are 1 and 2. (A solution for any other choice could be found with an argument similar to what follows.) Edge 9 is not in the cycle, hence, 5 and 6 both must be in. Now, one of 4 and 7 is in the cycle, and one of 3 and 8. Not both 7 and 8 can be in, however, or the cycle would contain the smaller cycle 1, 2, 7, 8. Also, as we saw in PAUSE 6, edge 3 and 4 cannot both be in the cycle. Hence, both 3 and 7 are in, or 4 and 8 are in. Assume that 3 and 7 are in. (The other case follows similarly.) We now have 7, 1, 2, 3, 6 and 5 in the cycle, while 9, 4, 8 are not in the cycle. Since 3 and 6 are in, edge 12, 13, 27 are not, so edges 11, 22, 14, 26, 19 and 20 are in. But now we have found a smaller cycle 7, 1, 2, 3, 6, 5, 14, 26, 20, 19, 22, 11 which does not contain all vertices. Hence, there is no Hamiltonian cycle. □