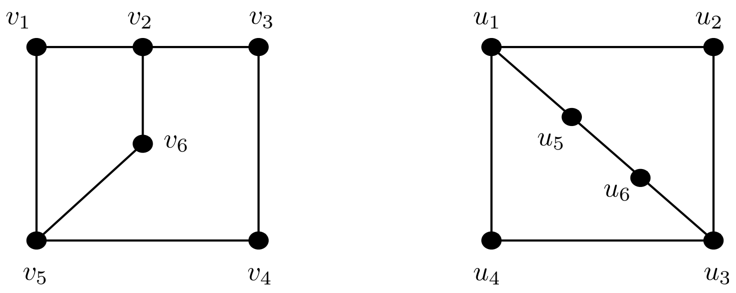
▶ **Problem 10.3-9**

(a) Find the adjacency matrices A_1 and A_2 of the graphs G_1 and G_2 shown.



(b) Explain why the function $\phi : G_1 \to G_2$ defined by

$$\phi(v_1) = u_4, \quad \phi(v_2) = u_1, \quad \phi(v_3) = u_5,$$

$$\phi(v_4) = u_6, \quad \phi(v_5) = u_3, \quad \phi(v_6) = u_2,$$

is an isomorphism.

(c) Find a permutation matrix P that corresponds to the isomorphism in (b) such that $PA_1P^T = A_2$.

Solution. (a)

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

(b) The function ϕ is an isomorphism because if the vertices of G_1 are relabeled, v_i being replaced by $\phi(v_i) = u_i$, then the adjacency matrix of G_1 relative to the u_i's is A_2. (See Theorem 10.3.3.)

(c)

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

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