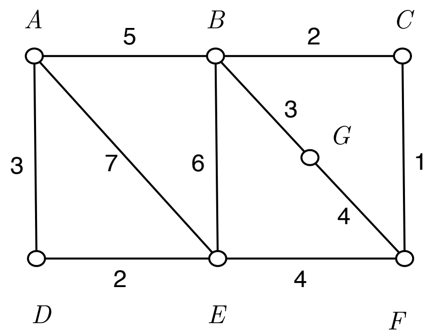
► Problem 12.1-9

The vertices in the graph represent towns; the edge, roads; and the labels on the roads, cost of paving the roads.

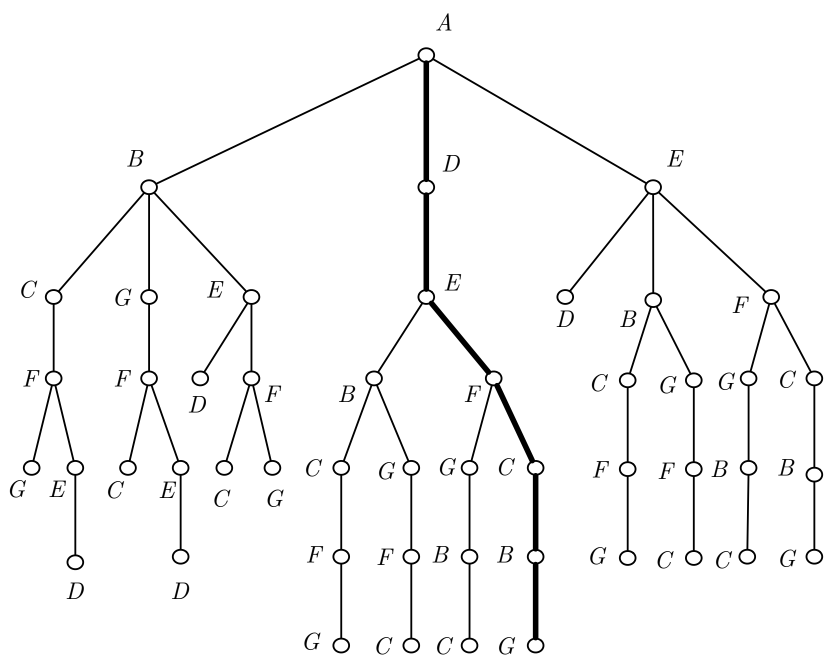


(a) Make a tree that shows all paths beginning at vertex A. List the paths that terminate at C. Indicate which, if any, are Hamiltonian.

(b) Is the graph Hamiltonian? Explain.

(c) Which roads should be paved so that one may drive from A along paved roads to as many towns as possible at minimal cost? Justify your answer. What is this minimal cost?

Solution. (a) The paths terminating at C are ABC, $ABGFC$, $ABEFC$, $ADEBC$, $ADEBGFC$, $ADEFGBC$, $ADEFC$, $AEBC$, $AEBGFC$, $AEFC$, and $AEFGBC$. The Hamiltonian paths are $ADEBCFG$, $ADEBGFC$, $ADEFGBC$, and $ADEFCBG$.



(b) No Hamiltonian paths end at a vertex adjacent to A, so the graph is not Hamiltonian. (Alternatively, vertices C and G of degree 2 impose a proper circuit.)

(c) Since there are Hamiltonian paths from A, we seek a paving of roads which allows us to visit every town with paved roads from A. To do this at minimal cost, we should pave the road along the Hamiltonian path $ADEFCBG$ at a cost of 15. This path is the best of the Hamiltonian paths, but it also must be cheaper than any other selection of edges connecting the six other towns to A. This is in fact the case, because $ADEFCBG$ consists of the six edges of smallest weight in the entire graph. $\square$